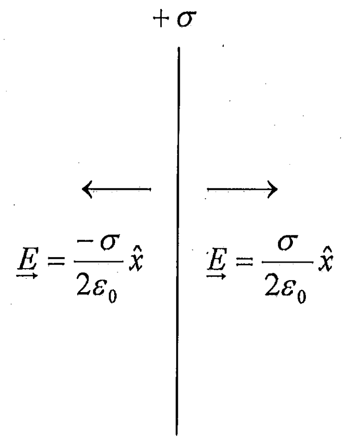
ELECTRIC POTENTIALS – Special Cases

1. Single plate having charge density $\sigma \; {}^{C}\!/\!{}_{m^2}$



$\Delta V = -\underset{\rightarrow}{E} \bullet \underset{\rightarrow}{\Delta S}$

Since $-\underset{\rightarrow}{E} \parallel \hat{x}$

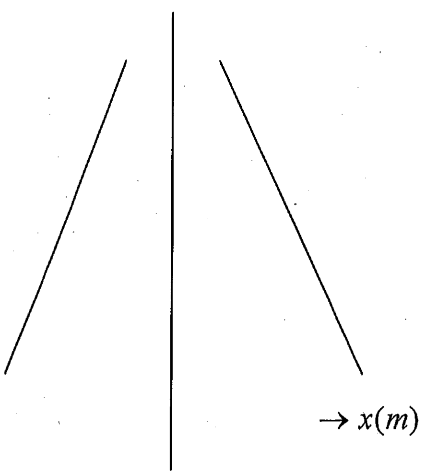
Nonzero ΔV only if $\Delta S \parallel \hat{x}$

$x > 0 \qquad \Delta S = x\hat{x}$

$\Delta V = -\frac{\sigma}{2\varepsilon_0}x$

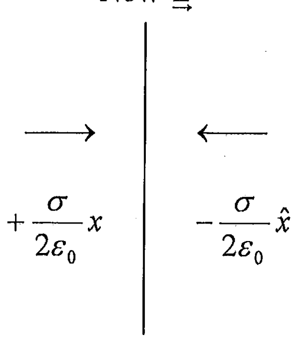
$x < 0 \qquad \Delta S = -x\hat{x}$

$\Delta V = \left(\frac{\sigma}{2\varepsilon_0}\right)(-x)$

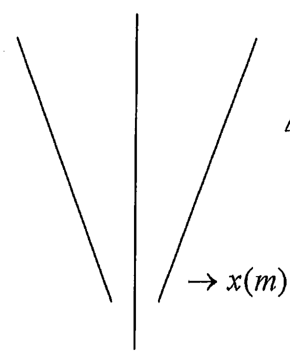


2. Single plate having charge density $-\sigma \; {}^{C}\!/\!{}_{m^2}$

Now $\underset{\rightarrow}{E}$



so



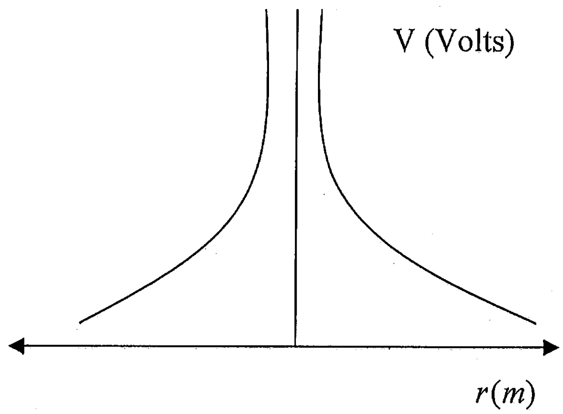
$\Delta V = \frac{\sigma}{2\varepsilon_0}x$

3. Single Point Charge Q at $r = 0$

$$\underset{\rightarrow}{E} = \frac{k_e Q}{r^2}\hat{r}$$

We will put $V = 0$ at large $r(r \to \infty)$ because E goes to zero at large r. Then calculate the change in V as we come from ∞ to r:

$$\Delta V = -\underset{\rightarrow}{E} \bullet \underset{\rightarrow}{\Delta r}$$



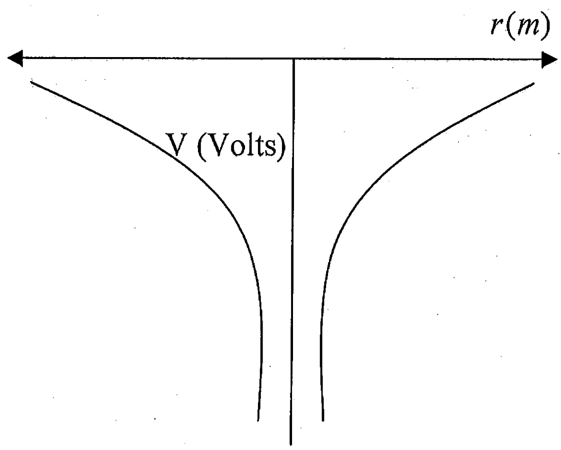
This requires an integral

$$V(r) = \frac{k_e Q}{r}$$

4. Single $-|Q|$ at $r = 0$

$$\underset{\rightarrow}{E} = \frac{-k_e Q}{r^2}\hat{r}$$

so $$V(r) = \frac{-k_e Q}{r}$$



5. Spherical shell or spherical conductor of radius R. In this case charge resides only on the surface hence

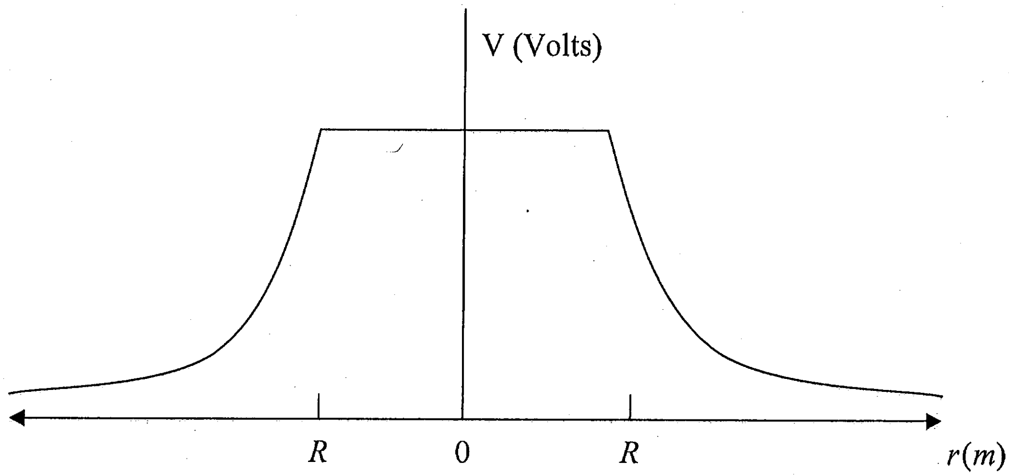
for $r < R$ $\underset{\rightarrow}{E} = 0$

for $r > R$ $\underset{\rightarrow}{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

The corresponding potential is

$$r > R \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

$$r < R \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 R}$$



6. Insulating sphere of radius R carries a charge Q distributed uniformly over the sphere so one can define a charge density

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3}$$

now

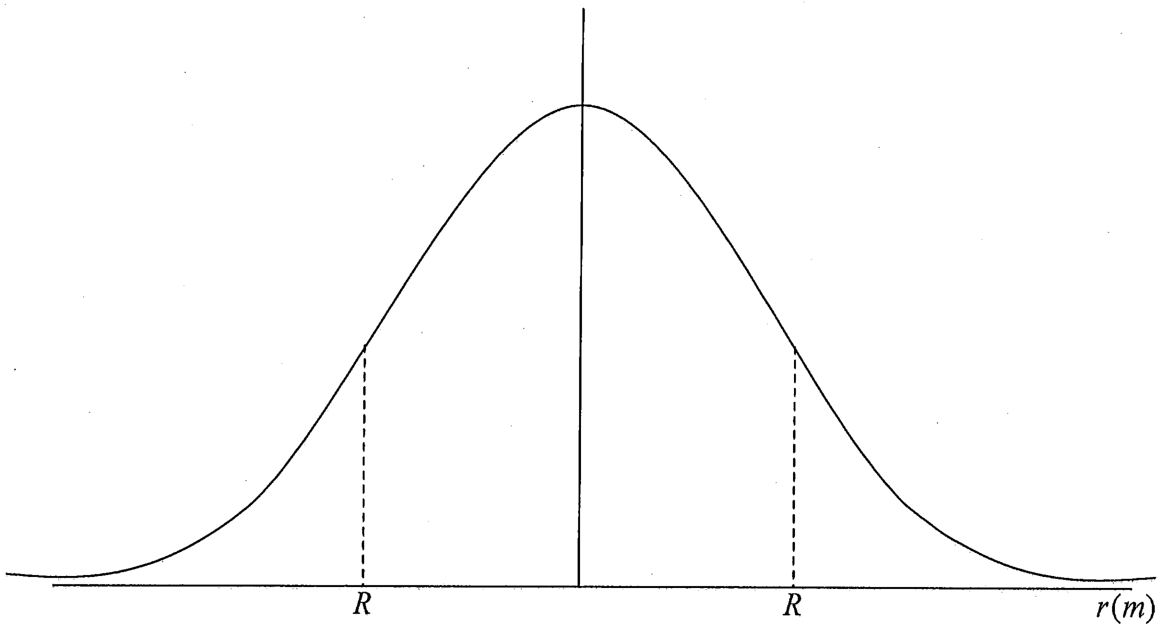
$$\text{for } r < R \qquad \underset{\rightarrow}{E} = \frac{\rho\, r}{3\varepsilon_0}\hat{r} = \frac{Q\, r}{4\pi\varepsilon_0 R^3}\hat{r}$$

$$\text{for } r > R \qquad \underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 R^2}$$

so

$$\text{for } r > R \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\text{for } r < R \qquad V(r) = \frac{Q}{4\pi\varepsilon_0 R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$



EQUIPOTENTIALS

Curves (in Two Dimensions) and surfaces (in Three Dimensions) where the Electric potential is constant [V =constant]. (You will do an experiment to trace equipotential curves) There are two important properties of an equipotential

(i) If a charge moves on an equipotential it will not cost any energy (reminder: it costs no work to move on a closed loop in a conservative force)

(ii) The $\underset{\rightarrow}{E}$ field must be perpendicular to an equipotential

Examples

(i) Plate carrying $+\sigma \ {}^{C}\!/_{m^2}$, $\Delta V = \frac{-\sigma}{2\varepsilon_0} x$

Equipotentials are planes parallel to plate

(ii) Pt. charge Q at $\varepsilon = 0$, $V(r) = \frac{k_e Q}{r}$

Equipotentials are spheres of radius r whose center is at $r = 0$

(iii) Surface of a conductor in stationary conditions – charge on surface only, $\underset{\rightarrow}{E} \perp$ surface everywhere so surface is equipotential.